Modal Analysis

ERA & DIAMOND

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ERA, the *Eigensystem Realization Algorithm*, is widely used as a modal analysis technique, generating a system realization using time-domain response input and output data. It was proposed by (Juang & Pappa,1985) [1].

Notes and MATLAB code based on (Caicedo,2011) [2].

Impulse Reponse

Impulse Response $h[k]$ was induced by equation

$$y[k] = h[k]p[0] \tag{1}$$

$y[k]$ is system reponse(the output) and $p[0]$ is impulse input. $h[k]$ is a $q \times p$ vector, for q outputs and p inputs.

Hankel Matrix

Hankel Matrix is a block matrix constituted by $h[k]$, and is shown below

$$\mathbf{H[k-1]} = \begin{bmatrix} h[k] & h[k+1] & \cdots & h[k+s-1] \\ h[k+1] & h[k+2] & \cdots & h[k+s] \\ \vdots & \vdots & & \vdots \\ h[k+r-1] & h[k+r] & \cdots & h[k+r+s-2] \end{bmatrix}_{(n_y \times r)\times(n_p \times s)} \quad (2)$$

- Dimension: $(n_y \times r) \times (n_p \times s)$
- $n_y \& n_p$: Outputs and Inputs
- $r \& s$: Chosen by programmer

SVD

Thin SVD to $H[0]$, obtained equation below

$$\mathbf{H[0]} = \mathbf{U}\Sigma\mathbf{V}^T \tag{3}$$

- $\mathbf{U}$: $(n_y \times r) \times n)$
- Σ: $n \times n)$
- $\mathbf{V}$: $(n_p \times s) \times n$.

n is the number of state variables.

Auxiliary Matrix $\mathbf{E_y}$ and $\mathbf{E_p}$

Constructing auxiliary matrix $\mathbf{E_y}$ and $\mathbf{E_p}$ as equation below

$$\mathbf{E_y} = \begin{bmatrix} \mathbf{I}_{n_y \times n_y} \\ \mathbf{0}_{n_y \times n_y} \\ \vdots \\ \mathbf{0}_{n_y \times n_y} \end{bmatrix}_{(n_y \times r) \times ny} \qquad \mathbf{E_p} = \begin{bmatrix} \mathbf{I}_{n_p \times n_p} \\ \mathbf{0}_{n_p \times n_p} \\ \vdots \\ \mathbf{0}_{n_p \times n_p} \end{bmatrix}_{(n_p \times s) \times np} \tag{4}$$

Inducing Matrix A B and G

Then we got matrix used in state-space representation in equation below

$$\text{State Matrix: } \mathbf{A} = \Sigma^{-1/2}\mathbf{U}^{\mathbf{T}}\mathbf{H}[\mathbf{1}]\mathbf{V}\Sigma^{-1/2}{}_{(n\times n)} \tag{5}$$

$$\text{Input Matrix: } \mathbf{B} = \Sigma^{1/2}\mathbf{V}^{T}\mathbf{E_p}{}_{(n\times p)} \tag{6}$$

$$\text{Output Matrix: } \mathbf{G} = \mathbf{E_y}^{T}\mathbf{U}\Sigma^{1/2}{}_{(q\times n)} \tag{7}$$

Modal Parameters

- Let $\boldsymbol{\Omega}$ be the eigenvalue of State Matrix $\mathbf{A}$
- Let $\mathbf{V}$ be the eigenvector of State Matrix $\mathbf{A}$

Let T be sampling period, we obtain

- Frequency ω: $\omega = |\ln(\boldsymbol{\Omega})/T|$
- Damping Ratio ζ: $\zeta = Re(\ln(\boldsymbol{\Omega})/T)/\omega$
- Vibration Modes: $\mathbf{V}$

DIAMOND

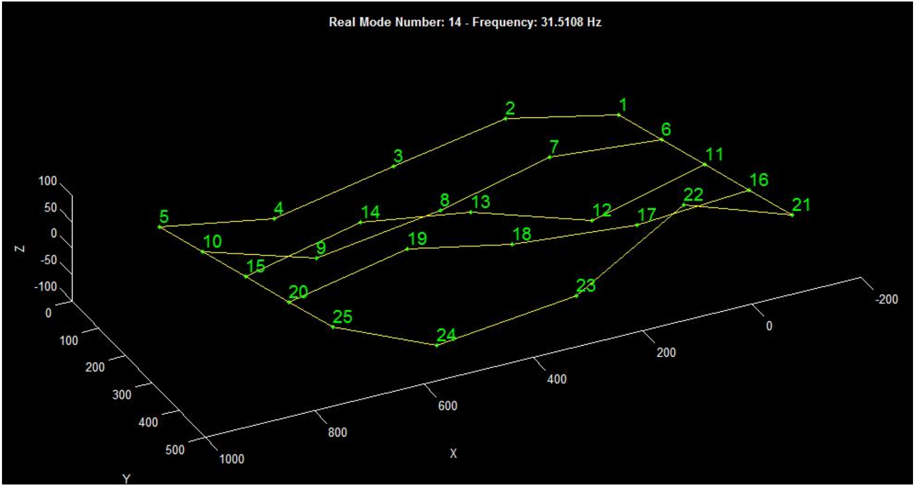
A MATLAB toolbox known as *Damage Identification And MOdal aNalysis for Dummies*

Developing by Los Alamos National Lab [2]

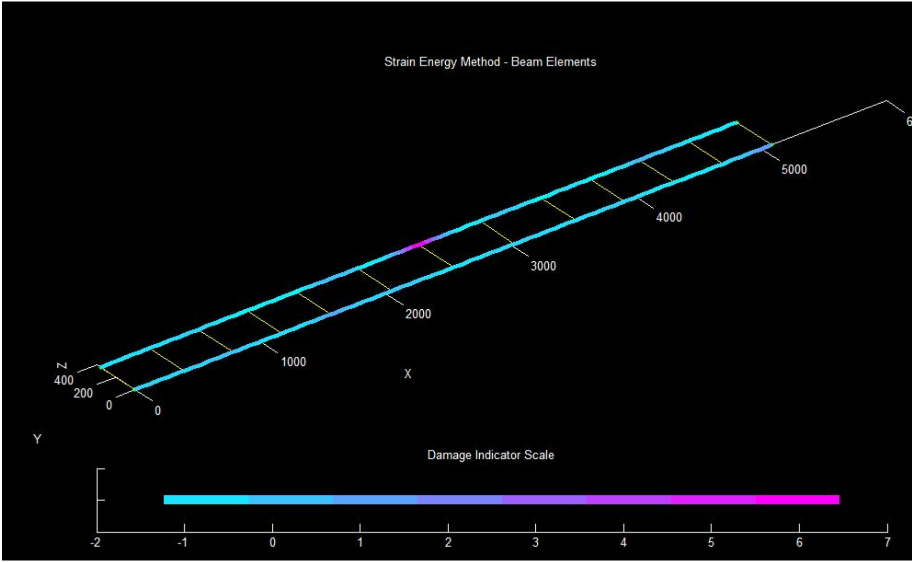
Overview of DIAMOND

- Modal Analysis
- Damage Identification
- Model Refinement
- Test Simulation

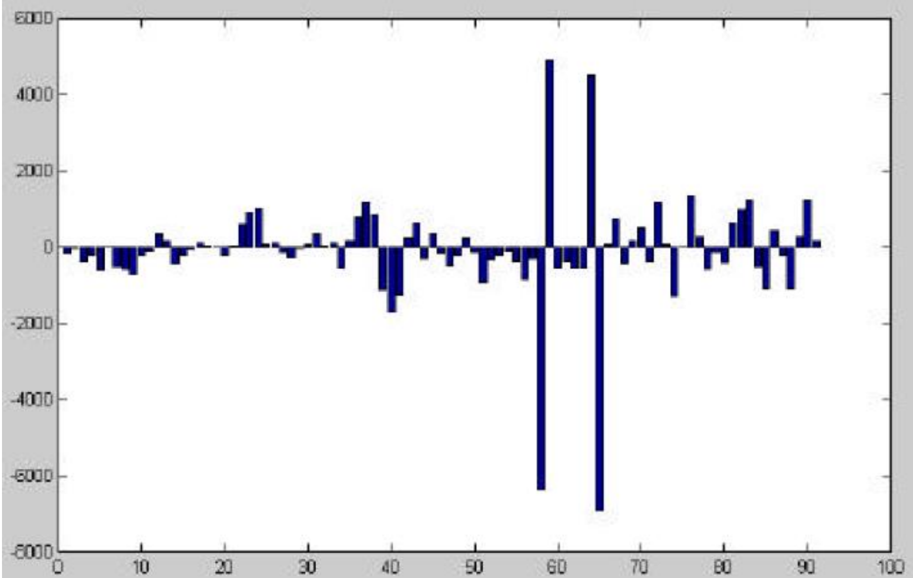
Modal Analysis



Damage Identification



Model Update Techniques



J-N Juang and Richard S Pappa.
An eigensystem realization algorithm for modal parameter identification and model reduction.
Journal of guidance, control, and dynamics, 8(5):620–627, 1985.

Juan M Caicedo.
Practical guidelines for the natural excitation technique (next) and the eigensystem realization algorithm (era) for modal identification using ambient vibration.
Experimental Techniques, 35(4):52–58, 2011.

Scott W Doebling, Charles R Farrar, and Phillip J Cornwell.
Diamond: A graphical interface toolbox for comparative modal analysis and damage identification.
In *Proceedings of the 6th International Conference on Recent Advances in Structural Dynamics, Southampton, UK*, pages 399–412, 1997.